2016 Portuguese Meeting on Optimal Control

Book of Abstracts

June 20th–21st, 2016

EPCO 2016

The 2016 Portuguese Meeting on Optimal Control – EPCO 2016 – will take place at the School of Engineering Polytechnic of Porto (ISEP) at Porto. EPCO 2016 will be held from Monday, 20th June to Tuesday, 21st June 2016 and it will provide an excellent opportunity for presenting new results and to discuss the latest research and developments in the field of optimal control. As in previous editions, this is an informal meeting seeking the exchange of knowledge and ideas among participants.

The meeting will cover a broad range of topics including:

- Application of optimal control to energy, medicine, robotics, economics, biology, etc,
- Optimization and Control Theories,
- Optimization Approaches to Control Synthesis,
- Stabilization methods for nonlinear systems,
- Model Predictive Control,
- Perturbed Systems,
- Differential Games, and
- Numerical Approaches and Solvers.

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Contents

Invited Speakers

Course **Chairman: Maria do Rosário de Pinho**

Understanding modern concepts of Optimization and Optimal Control with WORHPLab

June 20th 9:30
June 21st 9:00

Christof Büskens

University of Bremen

The ESA–NLP solver WORHP is already used in several academic and industrial projects in a wide range of applications, as aerospace, automotive or logistics. Currently over 600 users worldwide code their problem formulations using the standard interfaces to Fortran, C/C++, MATLAB and others.

To simplify the formulation of optimisation problems for demonstrational and educational purposes WORHPLab is developed as a graphical user interface (GUI). With a growing set of applied examples and visualisation techniques it shows the capabilities of the underlying solver WORHP and opens access to more involved concepts like parametric sensitivity analysis using WORHPZen and others.

Moreover, WORHPLab provides the possibility to solve optimal control problems using our transcription technique TransWORHP. Different approaches like full discretisation with grid refinement or multiple shooting are compared easily within this tool. Additionally, optimal control problems on reduced time horizons can be solved to illustrate concepts of non– linear model predictive control (MPC).

WORHP Lab was already employed successfully in several industrial workshops as well as for educational purposes with pupils and students. In this presentation, we try to bridge the gap between nonlinear optimization and optimal control problems by treating both: theory and practical implementation.

All participiants are invited to solve problems with the WORHPLab by themselves. A download adress will be provided in time.

Plenary Talk **Chairman: Fernando Lobo Pereira**

June 20th
14:40

Liapunov–like functions and Lie brackets

Franco Rampazzo
University of Padova

Under some controllability assumptions,the optimal time function is a particular Lyapunov function, a very efficient one indeed, for it also minimizes a cost, namely, the time to reach the target. In general, the optimal time function is not smooth, and this is somehow the price one has to pay for the high performance it guarantees. The situation is similar when the integral of a nonnegative current cost l replaces the time cost (which is the integral of $l = 1$). To pave the way towards an augmented regularity, we embed the standard dissipative relation in a differential inequality (DI) involving Hamiltonians built from the iterated Lie brackets of the dynamical vector fields. Actually, the solutions of (DI), besides yielding reachability of the target (in finite or infinite time), provide upper estimates for the minimum value function. Furthermore, because of the explicitly displayed controllability, solutions of (DI) can likely be expected more regular than the value function.

Talks

Talk Session 1 **Chairman: João Miranda Lemos**

Local Market Structure in a Hotelling Town

June 20th
11:30

Alberto A. Pinto[*,1], João P. Almeida[*,2], Telmo Parreia[3]

[*]LIAAD - INESC TEC
[1]Department of Mathematics, FCUP
[2]Polytechnic Institute of Bragança
[3]Department of Mathematics and Applications, University of Minho

For the quadratic Hotelling model, we study the optimal localization and price strategies under incomplete information on the production costs of the firms. We compute explicitly the pure Bayesian-Nash price duopoly equilibrium and we prove that it does not depend upon the distributions of the production costs of the firms, except on their first moments. We find when the maximal differentiation is a local optimum for the localization strategy of both firms.

Bayesian–Nash Equilibria in Theory of Planned Behavior

June 20th
11:50

João P. Almeida[*,1], Helena Ferreira[*], Bruno M.P.M. Oliveira[*,2], Alberto A. Pinto[*,3]

[*]LIAAD - INESC TEC
[1]Polytechnic Institute of Bragança
[2]FCNAUP
[3]Department of Mathematics, FCUP

We construct a model, using Game Theory, for the Theory of Planned Behavior and we propose the Bayesian–Nash Equilibria as one of many possible mechanisms to transform human intentions into behavior decisions. We show that saturation, boredom and frustration can lead to the adoption of a variety of different behavior decisions, as opposed to no saturation, which leads to the adoption of a single consistent behavior decision.

June 20th
12:10

High accuracy with Tau approximants for optimal control solutions

Alexandra Gavina[1], José Matos[1], Paulo Beleza Vasconcelos[2]

[1]Instituto Superior de Engenharia do Porto
[2]Faculdade de Economia da Universidade do Porto

Indirect methods to tackle optimal control problems (OCP), delivering optimal solutions by satisfying optimality conditions instead of minimizing a cost criterion directly as in direct methods, are often used. Usually, the approximate solution can only be obtained computationally requiring sophisticated numerical methods.

In this work we propose the use of spectral methods, namely the Tau method, to obtain the solution of the OCP through the associated boundary value problem BVP. The advantage of relying on spectral methods is that, if the problem has a smooth solution, they can ensure high accuracy.

The Tau method, initially developed to compute polynomial approximations to the solution of liner differential problems, is extended to compute approximate solutions of nonlinear differential problems. Novel in this work are the introduction of the linearization coefficients for the product of orthogonal polynomials, which numerically stabilizes the computation for moderate to high degree polynomial approximations.

Talk Session 2 **Chairman: Sofia Lopes**

The Principle of Maximum Entropy, from modeling to adaptive predictive control

June 20th
15:30

João M. Lemos[1]

[1]INESC-ID / Instituto Superior Técnico, Universidade de Lisboa

This presentation addresses the use of the Principle of Maximum Entropy (PME) for the modeling of physical processes from incomplete process data, to adaptive predictive control in the presence of un-modeled dynamics. In general [1], the problem consists of finding an unknown function given an incomplete set of facts that concern its properties. Since the function is not completely determined, the PME asserts that one should look for a function that complies with the known data, while maximizing the entropy. As such PME leads to a variational problem in which the functional to minimnize is the entropy and the known facts are constraints. The PME was introduced by Jaynes [2] in order to relate information theory to statistical mechanics. Burg, [1], made a seminal and well succeeded application of PME to high resolution spectral analysis, that led to the well known Burg method. Since then, PME was used in a variety of problems in signal processing and machine learning [3]. The application of PME to continuous time control and robotics was suggested by Saridis [4,5], with discrete time control applications described in [6]. The presentation will review the PME with emphasis on its relation with Variational Calculus and will show how the conclusions drawn from it have an impact on the design of adaptive predictive controllers that are able to tackle highly uncertain processes such as energy production plants, as described in [7].

[1] J.P. Burg (1975). Maximum Entropy Spectral Analysis. Ph. D. Thesis, Stanford University, 1975.
[2] E.T. Jaynes (1957). Infomation theory an statistica Mechanics. *Physical Review*, 106(4): 620-630.
[3] J. Kapur and H. Kesavan (1992). *Entropy optimization principles with applications.* Academic Press.
[4] G.N. Saridis (1988). Entropy formulation for optimal and adaptive control. IEEE Trans. Autom. Control, AC-33(8): 713-721.
[5] G.N. Saridis (1995). *Stochastic Processes, Estimation, and Control – The entropy approach.* John Wiley and Sons, Inc.
[6] Y. Tsai, F. Casiello and K. Loparo (1992). Discrete time entropy formulation of optimal and adaptive control problems. *IEEE Trans. Autom. Control*, 37:1083-1088.
[7] J.M. Lemos, R. Silva and J. Igreja (2014). *Adaptive Control of Solar Energy Collector Systems.* Springer.

This work was supported by FCT under the projects UID/CEC/50021/2013 and PTDC/EEI-PRO/0426/2014.

June 20th
15:50

On input–to–trajectory mappings of control systems with delays and impulses

Manuel Guerra

ISEG–Universidade de Lisboa and CEMAPRE–Centro de Matemática Aplicada à Previsão e Decisão Económica

It is well known that for multi–input control–afine systems of ODE's, involutivity of the controlled fields is a necessary condition for continuity of the input–to–trajectory map in the weak topology. This fact raises some difficulties to the construction of well–defined input–to–trajectory maps when the space of controls is large enough to include impulsive controls.

It turns out that continuity of the input–to–trajectory map with respect to the Fréchet metric in the space of controls and in the space of trajectories does not depend on any commutativity/involutivity assumption concerning the controlled fields. Thus, any smooth control afine system can be seen as a continuous mapping between spaces of Fréchet curves. The class of Fréchet generalized controls is broad enough to include impulsive controls and to allow for existence of minimizers for convex Lagrange variational problems of low (in particular, linear) growth.

In this paper, we show that some of the above results can be extended to controls systems where the dynamics include delayed effects. In particular, for a broad class of such systems, the input–to–trajectory map is still continuous with respect to the Fréchet metrics.

June 20th
16:10

Walrasian prices in random exchange markets

Bruno M.P.M. Oliveira[1,5], Yusuf Aliyu Ahmad[2], Athanasios N. Yannacopoulos[3], Barbel F. Finkenstädt[4], Alberto A. Pinto[2,5]

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[2]Faculdade de Ciências - Universidade do Porto
[3]Department of Statistics, University of Warwick, United Kingdom
[4]Department of Statistics, Athens University of Economics and Business, Greece
[5]LIAAD-INESC TEC, Porto, Portugal

We study a random matching economy, where pairs of participants are selected randomly to trade two goods at an agreed Edgeworthian price. We show that under some fairly general and easy to check symmetry conditions, depending on the initial distribution of endowments and the agents preferences, the sequence of Edgeworthian prices in this economy converges to the Walrasian price for this economy. Additionally, we also consider that each participant have a selfishness factor. This brings up a game alike the prisoner's dilemma, where may not be allowed, or where trade may occur in an asymmetric point in the core. We discuss how the selfishness affects the sequence of Edgeworthian prices.

This work is financed by National Funds through the FCT - Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) within project Dynamics, optimization and modelling PTDC/MAT-NAN/6890/2014.

Talk Session 3 **Chairman: Delfim F.M. Torres**

An optimal control approach to quantum splines

June 20th
17:00

Lígia Abrunheiro[1], Margarida Camarinha[2], Jesús Clemente-Gallardo[3], Juan C. Cuchí[4], Patrícia Santos[2,5]

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[2]CMUC, Department of Mathematics, University of Coimbra, Portugal
[3]Department of Theoretical Physics, BIFI and IQFR-BIFI, University of Zaragoza, Spain
[4]ETSEA, University of Lleida, Spain
[5]Polytechnic Institute of Coimbra / ISEC, Dept. of Physics and Mathematics, Portugal

The quantum splines are curves on the space of self–adjoint operators of a Hilbert space H, which generalize the notion of Riemannian cubic splines to the quantum domain. In quantum mechanics, physical magnitudes are represented by self–adjoint operators on a Hilbert space. For instance, if we consider the energy of a physical system, the associated operator is known as the Hamiltonian of the system. To define a quantum spline, it is considered a quantum control problem where the trajectory described by the quantum system must pass through a given set of points at a given set of times (or as close as possible), while keeping minimal the rate of change of the energy. The known analysis of this problem involves variational calculus on the Lie group $U(n)$ of unitary matrices. In this work, we present an optimal control approach to describe the quantum splines, using a geometrical formulation of Quantum Mechanics. As physical magnitudes are represented by self–adjoint operators on a Hilbert space we know that magnitudes associated with a finite dimensional Hilbert space can be identified with the unitary Lie algebra $u(n)$ and, via its canonical scalar product, with the elements of the dual space $u^*(n)$. The main goal here is to formulate an optimal control problem for a nonlinear system on $u^*(n) \times u^*(n)$ which corresponds to the variational problem of quantum splines. Moreover, the corresponding Hamiltonian equations and the interpolation conditions are derived. Our results are illustrated with some examples and the corresponding quantum splines are computed with the implementation of a suitable numerical method. Our formalism includes as particular cases the other formulations of the problem and offers the potential to extend it to more general dynamical situations, as the case of open systems.

An optimal control problem for a non autonomous prey–predator model in relevance to pest control

June 20th
17:20

Paulo Rebelo, Silvério Rosa, César M. Silva
Universidade da Beira Interior

The aim of this paper is to present and solve an optimal control problem for a non autonomous prey–predator model in relevance to pest control. This ecological system consists of a pest and its natural enemy, the predator. We also consider a non autonomous contact (periodic) rate, the role of infection to the pest population and the presence of some alternative source of food to the predator population.

June 20th
17:40

A new mixed integer optimal control formulation of UC Problem

Luís A.C. Roque[1], Dalila B.M.M. Fontes[2], Fernando A.C.C. Fontes[3]

[1]INESC–TEC, SYSTEC–ISR, Instituto Superior de Engenharia do Porto

[2]LIAAD–INESC–TEC, Faculdade de Economia, Universidade do Porto

[3]SYSTEC–ISR, Faculdade de Engenharia, Universidade do Porto

The Unit Commitment (UC) problem is a well–known combinatorial optimization problem in power systems. This work addresses a mixed integer optimal control formulation of the UC problem, with both binary–valued control variables and real–valued control variables. In the UC problem, the goal is to schedule a subset of a given group of electrical power generating units and also to determine their production output in order to meet energy demands at minimum cost. In addition, the solution must satisfy a set of technological and operational constraints.

This problem is usually formulated as a nonlinear mixed–integer programming problem and it has been solved in the literature by a large variety of optimization methods ranging from exact methods (such as dynamic programming, branch–and–bound) to heuristic methods (genetic algorithms, tabu search, simulated annealing, particle swarm). For medium sized power systems, exact methods can be used to solve the UC problem, successfully. However, for larger systems, the computation time of exact methods becomes impractical since the size of the solution space increases exponentially with the number of time periods and units in the system. In these cases, heuristic methods can be used to find near–optimal solutions.

In this work, we present a new formulation of the UC problem as a mixed–integer optimal control problem, with both binary–valued control variables and real–valued control variables. Then, we use a variable time transformation method to convert the problem into an optimal control problem with only real–valued controls. The optimal control problem is transcribed into a finite–dimensional nonlinear programming problem and it is solved using standard NLP solvers.

Optimal control applied to a viral marketing campaign

June 21st
11:00

João Gonçalves[1], Helena Sofia Rodrigues[2], M. Teresa T. Monteiro[1]
[1]University of Minho
[2]Instituto Politécnico de Viana do Castelo

The viral process of a communication marketing campaign through social networks can be modeled using epidemiological models. Companies and other institutions are interested in diffuse a message in short time, using limited resources. In this case a SIR (susceptible, infected, recovered) model is proposed to study the effects of a viral marketing strategy. Then, an optimal control problem is formulated in order to maximize the information spread. Some computational experiments are made. Finally, some conclusions are carried out and specific recommendations are proposed to marketeers.

A mathematical model for HIV/AIDS with application to Cape Verde

June 21st
11:20

Cristiana J. Silva[1], Delfim F.M. Torres[1]
[1]CIDMA, University of Aveiro

We prove the existence and uniqueness of a disease free and endemic equilibrium. The global stability of the equilibrium points is proven. Based on data provided by the Progress Report on the AIDS response in Cape Verde 2014, we calibrate our model to the cumulative cases of infection by HIV and AIDS from 1987 to 2012 and we show that our model predicts well this reality. A sensitivity analysis is done for the case study in Cape Verde.

Optimal control approach applied to depth of anesthesia

June 21st
11:40

Juliana Almeida[1], Teresa Mendonça[2], Paula Rocha[1]
[1]Universidade do Porto, Faculdade de Engenharia
[2]Universidade do Porto, Faculdade de Ciências

A state–feedback law to control the amount of hypnotic administered during a general anesthesia is proposed in this paper. The control law is obtained by solving an optimal control problem where a new simplified model is used to designed the control scheme. Due to the complexity associated to the positivity constrain in the input signal, an approximate solution is obtained by relaxed the original problem into a Semi-Definite Program. Moreover, to decrease the patient's variability an identification procedure is implemented based on the real patient's response.

June 21st
12:00

An Adaptive Mesh Refinement Algorithm for Model Predictive Control

Luís Tiago Paiva[1], Fernando A.C.C. Fontes[1]
[1]SYSTEC–ISR, Universidade do Porto, Faculdade de Engenharia

Model Predictive Control (MPC) is a technique that, by solving a sequence of open–loop optimal control problems, can generate state dependent (feedback) controls. The optimal predicted trajectories are iteratively updated based on measurements obtained at sampling instants. This leads to an intrinsic robustness that makes the method adequate to address disturbances or model-–plant mismatches.

We developed an adaptive time–mesh refinement algorithm providing local mesh resolution refining only where it is required. In this algorithm, we consider a time–dependent stopping criterion for the mesh refinement algorithm with different levels. In the end, the OCP is solved using MPC with an adapted mesh which has less nodes in the overall procedure, yet having maximum absolute local error of the same order of magnitude when compared against a refined mesh with equidistant–spacing.

[1] Luís Tiago Paiva, Fernando A.C.C. Fontes, Adaptive Time–Mesh Refinement in Optimal Control Problems with State Constraints. *Discrete and Continuous Dynamical Systems*, 35(9), pp. 4553–4572, September 2015.
[2] Luís Tiago Paiva. *Numerical Methods in Optimal Control and Model Predictive Control.* PhD thesis, PhD in Applied Mathematics, Universidade do Porto, December 2014. `http://hdl.handle.net/10216/77537`

June 21st
12:20

The irrigation systems in different fields with common reservoir

Sofia O. Lopes[1,2], Fernando A.C.C. Fontes[2]
[1]CMAT - Universidade do Minho
[2]SYSTEC–ISR, Universidade do Porto, Faculdade de Engenharia

Here, we make an overview of the recent results obtained on the study of the annual planning irrigation systems, LFPPG16. Namely, the study of the minimization of the water introduced in a reservoir to supply different fields with different types of crops where the water of precipitation can be collected in a given area, LF16.

[1] Sofia O. Lopes, Fernando A.C.C. Fontes, Rui M.S. Pereira, MdR de Pinho, M. Gonçalves. Optimal Control Applied to an Irrigation Planning Problem.Mathematical Problems in Engineering, Volume 2016 (2016), Article ID 5076879, 10 pages.
[2] Sofia O. Lopes, Fernando A.C.C. Fontes. Optimal Control for an Irrigation Problem with Several Fields and Common Reservoir, CONTROLO'2016 (accepted to publication).

Optimal control of a non–autonomous SEIRS model with vaccination and treatment

June 21st
14:40

Joaquim Mateus, César M. Silva, Delfim F.M. Torres, Paulo Rebelo, Silvério Rosa

Instituto Politécnico da Guarda

We study an optimal control problem for a non-autonomous SEIRS model with incidence given by a general function of the infectives, the susceptibles and the total population and with vaccination and treatment as control variables. We obtain existence and uniqueness results for our problem and, for the case of mass-action incidence we present some simulation results designed to compare an autonomous and corresponding periodic model and also the controlled and the corresponding uncontrolled model.

An optimal control model for the customer dynamics based on marketing policy

June 21st
15:00

Silvério Rosa, César M. Silva, Paulo Rebelo, Helena Alves, Pedro G. Carvalho

Universidade da Beira Interior
Instituto de Telecomunicações, Covilhã

We consider a compartmental model to study the best marketing strategy in a model for the evolution of the number of regular customers and referral customers in some corporation. This model, recently proposed by [1], is treated as an optimal control problem and the incentives policy are the control variables. We discuss existence and uniqueness of the solution of the optimal control problem. Some simulation is presented to validate the obtained results.

[1] A mathematical model for the customer dynamics based on marketing policy, César M. Silva, Silvério Rosa, Helena Alves and Pedro G. Carvalho, Applied Mathematics and Computation, Volume 273, 15 January 2016, pp. 42-53.

June 21st
15:20

Optimal Insurance, Consumption and Investment Decisions: A Duality Approach

Filipe Martins[1], Alberto A. Pinto[1], Diogo Pinheiro[2], Stanley R. Pliska[3]

[1]Faculty of Sciences, University of Porto and LIAAD-INESC.
[2]Brooklyn College of City University of New York (CUNY)
[3]University of Illinois at Chicago

In this work we analyse a consumption, investment and life insurance purchase problem, in a general model of a financial market with stochastic coefficients that we assume to be complete.

We use duality tools from convex analysis to deal with the non-Markovian problem of utility maximization, and we obtain optimal consumption, investment and life insurance purchase under very general utility functions. We analyse the case of deterministic coefficients, deducing a mutual fund result and the Hamilton-Jacobi-Bellman equation on that case, and we obtain explicit solutions for utility functions with constant relative risk aversion (CRRA).

June 21st
15:40

Applying Model Predictive Control to a solar furnace

Bertinho A. Costa[1], João M. Lemos[1]

[1]INESC-ID / IST / Universidade de Lisboa

This presentation describes the application of Model Predictive Control to a solar furnace, where concentrated solar energy is adjusted to perform high temperature material stress tests. This process has a nonlinear dynamics, that is caused by the temperature dynamics, that depends on the fourth power of temperature, and by the actuator, the shutter, that is nonlinear, which is employed to adjust the solar flux. Sun power variability due to weather conditions may affect the operation of a solar furnace and are compensated. Off-line identification is employed to characterize the process dynamics. Model predictive control with integral is formulated by minimizing an extended horizon quadratic cost function that penalizes deviations from a time varying target reference, and control variable increments.

The aim is to design a predictive controller that is able to track the temperature cycling profile without overshooting, to avoid melting the material sample, and to have a good response to disturbances. A distinguishing feature of this work is that it comprises active cooling to allow faster set-point decreasing. This feature results in a mathematical model of the process with different gains for positive and negative actions. An issue whose implications are discussed.

This work has been supported by the European projet SFERA-II and the FCT program UID/CEC/50021/2013.

Optimal control of a HIV model with delays

June 21st
16:00

Filipe Rodrigues, Cristiana J. Silva, Delfim F.M. Torres

Department of Mathematics, Center for Research and Development in Mathematics and Applications (CIDMA), University of Aveiro, 3810-193 Aveiro, Portugal

We propose a model for human immunodeficiency virus–type 1 (HIV–1) infection with intracellular delay and prove the local asymptotical stability of the equilibrium points. We introduce a control function representing the efficiency of reverse transcriptase inhibitors and consider the pharmacological delay associated to the control. We propose and analyse an optimal control problem with state and control delays. Through numerical simulations, optimal solutions are proposed for the minimization of concentration of virus and treatment costs.

June 21st
17:00

Regularity and optimality conditions for optimal control problems with geometric mixed constraints

A.V. Arutyunov[1], D.Yu. Karamzin[2], Fernando Lobo Pereira[2]
[1]Peoples' Friendship University of Russia
[2]University of Porto, Portugal

Consider the optimal control problem

$$\begin{cases} \text{Minimize} & \varphi(p) + \int_{t_1}^{t_2} f_0(x,u,t)dt \\ \text{subject to} & \dot{x} = f(x,u,t),\ t \in T, \\ & R(x,u,t) \in C, \\ & p \in K. \end{cases} \tag{1}$$

Here, $T = [t_1, t_2]$ is the time interval (which we assume fixed, and $t_2 > t_1$), $\dot{x} = \frac{dx}{dt}$, x is state variable, which takes values in the Euclidean space R^n, $p = (x_1, x_2)$ is the so called endpoint vector, where $x_1 = x(t_1)$, $x_2 = x(t_2)$, and $u(\cdot)$ taking values in R^m is the control function. The vector-function $R : R^n \times R^m \times R^1 \to R^r$ and the closed set C define the geometric mixed constraints. The control function $u(\cdot)$ is considered measurable and essentially bounded, such that, together with the arc $x(\cdot)$, satisfies the mixed constraints. The set K is closed and it defines the endpoint constraints which have to be satisfied as well. If the mixed constraints and the endpoint constraints are satisfied, then the control process (x, u) is called admissible. The control process (x^*, u^*) is called optimal, if the value of the minimizing functional at any admissible process is not less than its value at (x^*, u^*). For the classic formulation of the control problem, see [1].

The mappings in (1),

$$\begin{aligned} &\varphi : R^{2n} \to R^1, \\ &f : R^n \times R^m \times R^1 \to R^n, \\ &f_0 : R^n \times R^m \times R^1 \to R^1, \text{ and} \\ &R : R^n \times R^m \times R^1 \to R^r \end{aligned}$$

satisfy the following main hypothesis. The maps f, f_0, R are continuously differentiable in (x, u) for a.a. t. On any bounded set, these maps and their partial derivatives in (x, u) are bounded, Lebesgue measurable in t for all (x, u), and continuous in (x, u) uniformly in t. The scalar function φ is continuously differentiable.

Everywhere in what follows, assume that problem (1) has a solution (x^*, u^*).

Consider the set-valued map

$$U(x,t) := \{u \in R^m : R(x,u,t) \in C\}.$$

Definition 1 *A point $u \in U(x,t)$ is said to be regular provided that*

$$N_C(R(x,u,t)) \cap \ker \frac{\partial R^*}{\partial u}(x,u,t) = \{0\}. \tag{2}$$

Here, the set $N_C(y)$ designates the limiting normal cone in the sense of Mordukhovich, [2], and A^* denotes the conjugate matrix or operator A. The regularity of the point u means that the so called Robinson Constraint Qualification (RCQ) holds at u for the constraint system $R(x,u,t) \in C$, [3].

The condition (2) can be reformulated in the following way: there exists a number $\varepsilon > 0$ such that

$$\left| y \frac{\partial R}{\partial u}(x,u,t) \right| \geq \varepsilon |y|, \quad \forall\, y \in N_C(R(x,u,t)).$$

The upper bound of all such ε's is also known as *modulus of surjection* of the constraint system $M : R(x,u,t) \in C$. Let us denote the modulus of surjection to an arbitrary given constraint system $V : F(z) \in S$ at point z, by $\operatorname{sur} V(z)$.[1]

Then, the regularity of the point $u \in U(x,t)$ is equivalent to the relation

$$\operatorname{sur} M(x,u,t) > 0.$$

We denote by $U_{\text{reg}}(x,t)$ the subset of all regular points of $U(x,t)$. The subset of points for which $\operatorname{sur} M(x,u,t) \geq \varepsilon$ is denoted by $U^{\varepsilon}_{\text{reg}}(x,t)$. Note that this set may not be closed. It is clear that

$$\begin{aligned} U^{\varepsilon}_{\text{reg}}(x,t) &\subseteq U_{\text{reg}}(x,t) \subseteq U(x,t) \;\; \forall\, \varepsilon > 0, \text{ and} \\ U^{\alpha}_{\text{reg}}(x,t) &\subseteq U^{\beta}_{\text{reg}}(x,t) \;\; \text{for } \alpha > \beta > 0, \end{aligned}$$

and $U^{0}_{\text{reg}}(x,t) = U(x,t)$.

The following concept corresponds to the classic approach to regularity for mixed constraints. (The so-called strong regularity.)

Definition 2 *The trajectory $x^*(t)$ is said to be regular w.r.t. the mixed constraints provided there is a number $\varepsilon_0 > 0$ such that*

$$U(x^*(t),t) \subseteq U^{\varepsilon_0}_{\text{reg}}(x^*(t),t), \;\textit{for a.a.}\; t \in T.$$

However in what follows a weaker regularity condition will be used.

Definition 3 *The trajectory $x^*(\cdot)$ is said to be weakly regular w.r.t. the mixed constraints provided there is a number $\varepsilon_0 > 0$ such that*

$$u^*(t) \in U^{\varepsilon_0}_{\text{reg}}(x^*(t),t) \;\; \textit{for a.a.}\; t \in T.$$

[1] In the literature, the modulus of surjection is introduced for set-valued maps $G : X \to 2^Y$. If spaces X, and Y are finite dimensional, then

$$\operatorname{sur} G(x|y) = \inf\{|x^*| : x^* \in D^*G(x,y)(y^*),\ |y^*| = 1\}.$$

Here, $D^*G(x,y)$ is the limiting coderivative of G at (x,y). By definition, $\operatorname{sur} G(x|y) = \infty$ when $y \notin G(x)$. If we set $G(\cdot) := R(x,\cdot,t) - C$, then $\operatorname{sur} M(x,u,t) = \operatorname{sur} G(x,u,t|0)$.

The regularity condition imposed in Definition 3 is weaker than the one from Definition 2, as it holds only locally in a small tube about $u^*(t)$, but not for all feasible points. The price to pay for this sharp drop down from the global to the local nature is the modified Weierstrass-Pontryagin maximum condition (6) that it appears in Theorem 1. See the discussion in [4] for more details and examples over the given concepts.

Along with the regularity, we also need the notion of the proper point. Let us introduce it. Let δ be a positive number and $u_0 \in U(x,t)$. Along the constraint system M defining the mixed constraints in problem (1), consider the associated constraint system

$$M_{\delta,u_0}: \quad \begin{cases} R(x,u,t) \in C, \\ |u-u_0| \leq \delta. \end{cases}$$

Definition 4 *A point $u_0 \in U(x,t)$ is said to be proper (or, α,γ-proper) provided there exist $\alpha,\gamma > 0$ such that*

$$\operatorname{sur} M_{\delta,u_0}(x,u,t) \geq \gamma \ \ \forall\, u \in U(x,t): \ |u-u_0| \leq \delta, \ \ \forall\, \delta \in (0,\alpha).$$

Results of [4] suggest a large subclass of the constraint systems for which any regular point is proper. Such a subclass includes convex sets, semi-algebraic sets, or even more general than semi-algebraic type of the sets, the sets which admit the so-called Whitney stratification, i.e., satisfying the Whitney condition b).

Let us impose the following condition.

Condition P) For all $\varepsilon > 0$, $\exists\, \gamma > 0$ such that, for any measurable bounded selector $u(t)$ of the map $U^\varepsilon_{\text{reg}}(t) := U^\varepsilon_{\text{reg}}(x^*(t),t)$, there exists a measurable scalar function $\alpha(t)$ s.t. $u(t)$ is $\alpha(t),\gamma$-proper for a.a. t.

Condition P) may seem somewhat cumbersome, but this condition is satisfied for the above mentioned subclass of the constraint systems. This means that the result following below is valid under C convex, or semi-algebraic, or, even, when the set C admits Whitney stratification.

Following [1], we introduce the Hamilton-Pontryagin function

$$H(x,u,t,\psi,\lambda) = \langle \psi, f(x,u,t)\rangle - \lambda f_0(x,u,t).$$

Under the weak regularity condition the following theorem is true.

Theorem 1 (Maximum Principle) *Let $\varepsilon \in (0,\varepsilon_0)$. Suppose that the process (x^*,u^*) is optimal to problem (1), the arc $x^*(t)$ is weakly regular w.r.t. the mixed constraints and that Condition P) is satisfied.*

Then, there exist a number $\lambda \geq 0$, an absolutely continuous function $\psi: T \to R^n$, an essentially bounded measurable function $\eta: T \to R^r$, and a constant $\kappa > 0$, which all depend on ε, such that

$$\eta(t) \in \operatorname{conv} N_C(R(t)) \ \ \textit{for a.a.}\, t, \tag{3}$$

$$\dot{\psi}(t) = -\frac{\partial H}{\partial x}(t) + \eta(t)\frac{\partial R}{\partial x}(t) \ \ \textit{for a.a.}\, t, \tag{4}$$

$$(\psi(t_1), -\psi(t_2)) \in \lambda \frac{\partial \varphi}{\partial p}(p^*) + N_K(p^*), \quad (5)$$

$$\max_{u \in \mathrm{cl}\, U^{\varepsilon}_{\mathrm{reg}}(t)} H(u,t) = H(t) \ \ \textit{for a.a.}\ t, \quad (6)$$

$$\frac{\partial H}{\partial u}(t) - \eta(t) \frac{\partial R}{\partial u}(t) = 0 \ \ \textit{for a.a.}\ t, \quad (7)$$

$$|\eta(t)| \leq \kappa(\lambda + |\psi(t)|) \ \ \textit{for a.a.}\ t, \quad (8)$$

$$\textit{and} \qquad \lambda + |\psi(t)| > 0 \ \ \forall\, t \in T. \quad (9)$$

Here, if some of the arguments of a function or of a set-valued map are omitted, then it means that the extremal values $x^*(t)$, $u^*(t)$, $\psi(t)$, and λ are in the place of the omitted arguments.

This result covers the corresponding results from [5], where C was considered merely convex.

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Lipschitzian regularity of solution to a problem of calculus of variations with quadratic integrand

June 21st
17:20

Miguel Oliveira[1], Gueorgui Smirnov[1]

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We obtain explicit estimate for Lipschitz constant of solution to a problem of calculus of variations with quadratic integrand. The approach is based on transformation of the problem into a time-optimal control problem, suggested by Gamkrelidze. This estimate can be used to obtain complexity bounds for numerical optimization methods.

June 21st
17:40

On the sufficiency of Pontryagin's maximum principle

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Traditionally, sufficient conditions of optimality involve second order derivatives. This is still the case when we are dealing with optimal control problems. However, for certain classes of these problems the Pontryagin maximum principle is by itself a sufficient condition. This is well known for linear control problems with convex cost and convex constraints.

Here, we introduce a *refined maximum principle condition* that for possible non convex problems with affine control systems and polyhedral set of controls, guarantees weak local optimality of control processes. This refined maximum principle condition means that the control is uniquely defined for almost all instants of time and the behavior of adjoint variables is rather regular. We illustrate this sufficient condition with different examples.

Posters

Optimal control for Epidemiology: choice of cost and introduction of constraints

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Our aim is to study the control of the spreading of infectious diseases when treatment and vaccination are introduced. We consider SEIR and SIR models to model the dynamics of the infections under consideration. The choice of certain treatment and vaccination policies is conditioned by what is taken into consideration. In optimal control terms, this means taking into account different costs. Here we consider different costs and constraints and we compare the economic relevance of each choice of cost. From the point of view of optimal control, this is also of interest since we illustrate how the cost affects the profiles of optimal control.

Approximate equilibria for a T cell and Treg model

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We analyse a model of immune response by T cells (CD4), where regulatory T cells (Tregs) act by inhibiting IL-2 secretion. We introduced an asymmetry reflecting that the difference between the growth and death rates can be higher for the active T cells and the active Tregs than for the inactive T cells and inactive Tregs. This asymmetry mimics the presence of memory T cells. In this paper we start by analysing the model in the absence of Tregs. We obtain an explicit formula that gives approximately the antigenic stimulation of T cells from the concentration of Tregs. Afterwards, we present an explicit formula that describes approximately the balance between the concentration of T cells and the concentration of Tregs; and an explicit formula that relates approximately the antigenic stimulation of T cells, the concentration of T cells and the concentration of Tregs. For our parameter values,

the relation between the antigenic stimulation of T cells and the concentration of T cells is an hysteresis that is unfold when some of the parameters are changed. We also consider a linear tuning between the antigenic stimulation of T cells and the antigenic stimulation of Tregs. Again, we have obtained an explicit formula relating approximately the antigenic stimulation of T cells, the concentration of T cells and the concentration of Tregs. With it, we can explain the appearance of an isola and a transcritical bifurcation.

Cournot duopolies with R&D investment in the optimal reduction of production costs

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We study Cournot models where two firms compete in a duopoly. In the first stage of the game, these firms can make investments in R&D to reduce the production costs in each period of the game. In the second stage, after the cost reduction, the firms choose their optimal output quantities. Firms select their strategies in order to maximize their profit. We will use the usual cost reduction function in literature introduced by D'Aspremont and Jacquemin and the cost reduction function introduced by Ferreira et al.. We aim to study the effect of the initial production costs. We will study the effect of the current production cost in the optimal investment, in the resulting quantities and profits. Special attention will be given to regions with multiple Nash equilibria.

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Optimal Control of a Furuta Pendulum

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The Furuta pendulum is a rotational pendulum that is actuated at its basis by a direct current motor with a gear. Two control problems associated to it consist of swinging-up the pendulum, in order to move it from the downwards position up to the upwards one, and then to equilibrate the pendulum in the upwards position. In both cases, optimal control methods are used. The swing-up problem is solved by formulating it as an optimal control problem with a convenient cost, that is then solved by using a numerical method to approximate the Pontryagin's necessary conditions. The numerical method relies on the iterative solution of the state

equation (forwards), of the adjoint equation (backwards), and on the optimization of the hamiltonian function with respect to the manipulated variable in a grid of time points.

Different aspects related to this problem are considered, that include the selection of an appropriate cost and numerical procedure details. The equilibration problem is solved with a standard LQG controller that is activated within a region of the state-space that is close to the upwards, zero velocity, state. A numerical study of the attraction region of the LQG equilibrating controller is performed, in order to show that this controller will fulfill its objective, even in the presence of a saturation non-linearity in the actuator. The algorithms used are described, as well as simulation and experimental results. The poster will be complemented with a short movie that shows the controller described applied to a real pendulum.

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Vaccination Games

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In the case of voluntary vaccination, people have to decide if the benefits of vaccination outweigh the adverse effects that may result from vaccination. The decision depends on the morbidity risks from vaccination and from infection, but also depends on the decision of the other individuals.

In this talk, we will make a game theoretical analysis of this vaccination game to find the vaccination strategy that maximizes an individual's payoff. Using the classical SIR epidemic model, the optimal strategy is simple and unique. Considering the SIRI model, by introducing reinfection in the SIR model, we observe the existence of multiple optimal strategies for the same level of the morbidity risks. This study shows that the impact of vaccination scares can be much more devastating in the presence of the reinfection. Also, the vaccination campaigns might not be as efficient as in the absence of reinfection.

Towards a MPC for open water systems considering both quantitative and qualitative aspects

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Model predictive control (MPC) is a well known technique for hydraulic structures control in open water systems. Many examples of its application are implemented considering quantitative aspects, like water levels in reservoir and canals or discharges at turbines or pumps.

Our aim is to develop and implement a model predictive controller for hydraulic structures of reservoirs considering water quality aspects of canals, rivers or reservoirs. An example, based on a drainage system of a polder composed by several connected canals will be presented. We use model predictive control in order to control the level of water in the polder canals. In this problem, some restrictions should be taken into account. More concretely, the water level should not exceed a certain maximum value, it should not be lower than a certain minimum value, and the capacity of the pump flow is limited. We simulate the water level controller performance for several scenarios, and weather conditions, and we compare the results with other control techniques.

The rational to further implement qualitative aspects will be presented.

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