

Title Liapunov-like functions and Lie brackets

Franco Rampazzo

Abstract Under some controllability assumptions, the optimal time function is a particular Lyapunov function, a very efficient one indeed, for it also minimizes a cost, namely, the time to reach the target. In general, the optimal time function is not smooth, and this is somehow the price one has to pay for the high performance it guarantees. The situation is similar when the integral of a nonnegative current cost l replaces the time cost (which is the integral of $l = 1$). To pave the way towards an augmented regularity, we embed the standard dissipative relation in a differential inequality (DI) involving Hamiltonians built from the iterated Lie brackets of the dynamical vector fields. Actually, the solutions of (DI), besides yielding reachability of the target (in finite or infinite time), provide upper estimates for the minimum value function. Furthermore, because of the explicitly displayed controllability, solutions of (DI) can likely be expected more regular than the value function.